Born level bound states*

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Even a first approximation of bound states requires contributions of all powers in the coupling. This means that the concept of "lowest order bound state" needs a proper definition. I discuss the "Born" (no loop, lowest order in $\hbar$) approximation. Born level states are bound by gauge fields which satisfy the classical field equations. In the rest frame, Gauss' law determines a distinct Coulomb field $A^0(\boldsymbol{x})$ for each instantaneous position of the charges. In frames where the bound state is in motion the gauge field is obtained by a Lorentz boost of the rest frame field.

As a check of the method, Positronium states are determined as eigenstates of the QED Hamiltonian, quantized at equal time. In the rest frame the relativistic bound state equation reduces to the Schrödinger equation in the $\alpha \to 0$ limit. In a general frame the energy eigenvalue has the correct dependence on the bound state momentum, and the wave function Lorentz contracts in agreement with classical relativity.

Electron states bound by a strong external field $A^\mu(\boldsymbol{x})$ are similarly determined as eigenstates of the Dirac Hamiltonian. The bound states have Fock components with any number of e^+e^- pairs, whose distribution is specified by the Dirac wave function. A linear potential confines electrons but repels positrons, which excape to large separations with correspondingly high energies. This clarifies why the Dirac mass spectrum is continuous for polynomial potentials, and the Dirac wave functions have both bound state and scattering characteristics.

The classical solutions of Gauss' law are explored also for hadrons in QCD. A non-vanishing boundary condition at spatial infinity generates a constant $\mathcal{O}\left(\alpha_s^0\right)$ color electric field between quarks of specific colors. Poincaré invariance limits the solutions to color singlet $q\bar{q}$ and qqq states, which do not generate an external color field. This restricts the $\mathcal{O}\left(\alpha_s^0\right)$ interactions between hadrons to string breaking dynamics as in dual diagrams. Light mesons are found to lie on linear Regge and parallel daughter trajectories. There are massless states which may be significant for chiral symmetry breaking. Since the bound states are defined at equal time in all frames they have a non-trivial Lorentz covariance.

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*P. Hoyer, *"Lectures on Bound states"*, arXiv:1605.01532 [hep-ph].