

Scattering asymptotic conditions in Euclidean relativistic quantum theory

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The locality axiom of the Osterwalder-Schrader axioms [1] of Euclidean quantum field theory is logically independent of the remaining axioms. This opens the possibility of formulating relativistic quantum mechanical models directly in a Euclidean representation. There are several reasons for considering this. One, locality is the source of all of the mathematical difficulties of quantum field theory. Second, lattice and Schwinger-Dyson formulations of QCD are most naturally formulated in a Euclidean representation. Third, cluster properties, which are difficult to realize in ordinary relativistic quantum mechanical models are easily satisfied in the Euclidean framework. Fourth, the Euclidean formulation of relativistic quantum mechanics has a direct connection to a light-front representation.

The Euclidean formulation has a different set of difficulties. There is the problem of analytic continuation to real time. This requires knowing the analytic structure of the Green functions. It is also necessary to ensure that computational errors remain small after continuation. A second difficulty is the requirement that the Green functions satisfy the condition of reflection positivity, which is necessary for a quantum mechanical interpretation.

One of the consequences of the Osterwalder-Schrader reconstruction theorem is that as long as one satisfies reflection positivity, it is possible to reconstruct a relativistic quantum theory without explicit analytic continuation. This allows one to avoid the need for explicit analytic continuation and potential errors that might arise from analytic continuation. This Osterwalder-Schrader construction gives the physical Hilbert space and a set of Poincaré generators that satisfy cluster properties, all expressed directly in terms of Euclidean Green functions and Euclidean test functions.

In principle, as long as the cluster properties are strong enough, this framework is in principle sufficient to formulate a mathematically well-defined scattering theory.

In theories like QCD, it is necessary to deal with scattering involving composite asymptotic states. In this talk I discuss the formulation and convergence of the scattering asymptotic conditions with composite particles in the Euclidean framework [2]. The problem has interesting connections to the Stiltjes moment problem in classical analysis, and various results on completeness of orthogonal polynomials on the half line.

[1] K. Osterwalder and R. Schrader, *Commun. Math. Phys.* 31, 83 (1973).

[2] Gordon Aiello and W. N. Polyzou, *Scattering asymptotic conditions in Euclidean relativistic quantum theory*, *Phys. Rev. D*93 (2016), arXiv:1512.03651.